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Short note

Multifractal thermodynamics of hadron-hadron and hadron-nucleus interactions

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Abstract. It is shown that the constant-specific-heat approximation is applicable to multifractal thermodynamics of hadron-hadron and hadron-nucleus interactions at high energies. Moreover, the constant specific heats calculated from experimental data on hadron-hadron and hadron-nucleus interactions have approximately the same value for both these types of multifractal multiproduction. Thus this parameter may turn out to be an universal characteristic of the hadron-hadron and hadron-nucleus interactions. Some relationship of this phenomenon to multifractal thermodynamics at the onset of chaos is briefly discussed.

PACS. 13.85.Hd Inelastic scattering: many-particle final states

Since papers [1] intermittency of multiparticle production is one of the intensively developing fields of hadron physics (see for recent reviews [2], [3] and references therein). Now there are numerous experimental evidences of the intermittent behavior obtained from hadron-hadron, hadron-nucleus, and nucleus-nucleus interactions. Interpretation of these experimental data is still an actual problem. It is known, in particular, that the multifractality can be considered from a thermodynamic point of view [2], [4], [5]. This interpretation is usually used to study the phase-transition like phenomena in the intermittent data [4], [6], [7]. On the other hand, some simple thermodynamic approximations could be applicable to the multiparticle production intermittency as well. In this note we will show that the constant-specific-heat (CSH) approximation (widely used in the ordinary thermodynamics [8]) is applicable to the multifractal data. Moreover, the constant specific heats calculated from a few sets of data corresponding to different types of interactions have approximately the same value and, therefore, may turn out to be an universal characteristic of the multiparticle production at high energies.

It should be noted, that the regions of the temperature where the constant-specific-heat approximation is applicable are usually far away from the phase-transition regimes. In particular, one can conclude from comparison with some experimental data studied in this note, that for the reactions considered here the phase-transition like phenomena could occur in a vicinity of point $q = 0$ only (where q is an *inverse* multifractal temperature). So that, on the temperature axis the distance between the constant specific heat regime and the phase-transition regime can be rather large. A fractal implication of this phenomenon is briefly discussed at the end of this Short Note.

Let us recall some standard definitions. Let $\Delta\eta$ be the pseudorapidity interval, and subdivide into M bins each of width $\delta\eta = \Delta\eta/M$. Let N be the number of particles in one event in $\Delta\eta$ interval and k_m be the number of particles in the m-th bin. The G_q moments are defined as

$$G_q = \sum_{m=1}^{M} p_m^q$$

where $p_m = k_m/N$ is the probability of particles in the m-th bin for one event and q is any real number. The summation is carried out over non-empty bins only. If the particle production process exhibit self-similar behavior then the moment follow the power law

$$G_q \propto (\delta\eta)^{\tau(q)}.$$

The generalized dimension spectrum is then given by:

$$D_q = \tau(q)/(q-1). \qquad (1)$$

One can also describe multifractality with so-called $f(\alpha)$ distribution [5] (and references therein). The $\tau(q)$ is related to $f(\alpha)$ by Legendre transforming

$$\tau(q) = q\alpha - f(\alpha), \qquad \frac{df}{d\alpha} = q \qquad (2)$$

The thermodynamics interpretation of these relationships means that q can be interpreted with an inverse temperature $q = T^{-1}$, the spectrum $f(\alpha)$ and α play the role

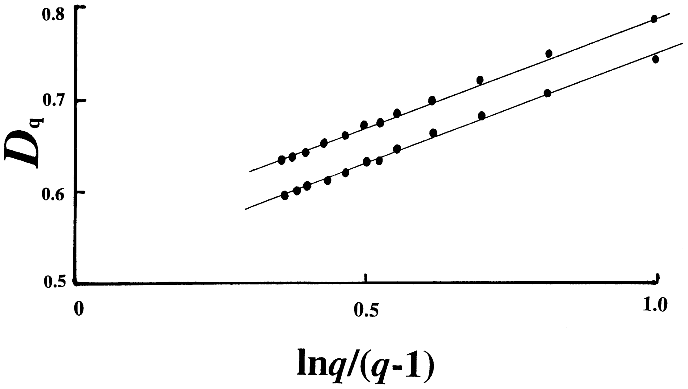

Fig. 1. The generalized dimensions D_q against $\ln q/(q-1)$ for interaction of hadrons with emulsion nuclei (adapted from [10]). The straight lines are drawn for comparison with CSH-approximation (4). Lower set of dots corresponds to $N_g = 0$ and upper set of dots corresponds to $N_g = 1, 2$

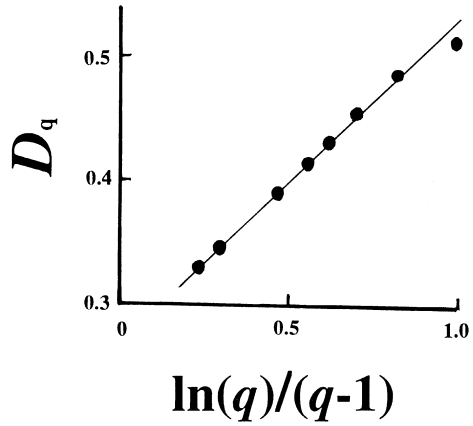

Fig. 2. The generalized dimensions D_q against $\ln q/(q-1)$ for π^+p interactions (adapted from [12]). The straight line is drawn for comparison with CSH-approximation (4)

of the entropy and the energy (per unit of volume) correspondingly [2], [4], [5].

It is well known that in usual thermodynamics in many important cases the specific heat of gases and solids is constant, independent of temperature, over a greater or smaller temperature interval [8]. In this case the entropy is given by following approximation [8]

$$f(q) = a - c \ln q \tag{3}$$

where a and c are some constants, and c is the constant specific heat. Concerning the constant a one can see from the CSH-approximation for the generalized dimensions

$$D_q \simeq (a-c) + c\frac{\ln q}{(q-1)}. \tag{4}$$

(corresponding to (3)) that $a \simeq D_1$. It is known (see, for instance, [9] and references therein) that the *information* dimension D_1 is the exponent of scaling of the number of bins containing the dominant contribution to the total "mass" and the name - "information" dimension, is chosen for this quantity due to relationship

$$\sum_m p_m \ln p_m \simeq D_1 \ln(\delta\eta).$$

Since the information dimension is by a natural way sensitive to differences in considered processes one cannot expect that the constant a takes an universal value. On the other hand, one can expect that the thermodynamic constant - c, can exhibit some universality. And indeed, in a recent paper [10] the multifractal spectra have been constructed using experimental data [11] on interaction of hadrons (a proton beam of energy 800 GeV/c) with emulsion nuclei. The multifractal spectra were constructed in [10] for various cuts on the number (N_g) of medium-energy ("grey") particles. We show in Fig. 1 (adapted from [10]) the data corresponding to $N_g = 0$ (lower set of dots) and to $N_g = 1, 2$ (upper set of dots). The straight lines are drawn for comparison with the CSH-approximation (4). One can also calculate the multifractal specific heat $c \simeq 0.25$ for both sets of the experimental data.

In Fig. 2 (adapted from [12]) we show analogous experimental data reported by the NA22 collaboration [13] who investigated the π^+p interactions with the centre-of-mass energy $(s)^{1/2} = 22GeV$. The straight line is drawn for comparison with the CSH-approximation (4). The multifractal specific heat $c \simeq 0.26$ calculated from this Fig. is approximately the same as those calculated from Fig. 1. Thus this parameter may turn out to be an universal characteristic of the multiparticle production. To obtain this value of the multifractal specific heat from the first principles (see, for instance, [2], [3]) seems to be an interesting problem for future investigations.

It should be also noted, that from Figs. 1 and 2 one can conclude that for the considered reactions the phase-transition like phenomena could occur in a vicinity of point $q = 0$ only. It means, in particular, a topological reconstruction. Similar situation is also naturally expected at the onset of chaos of the dynamical attractors. And indeed, in Fig. 3 we show generalized dimensions corresponding to a dynamical attractor in phase space constructed from the velocity fluctuation measured in the

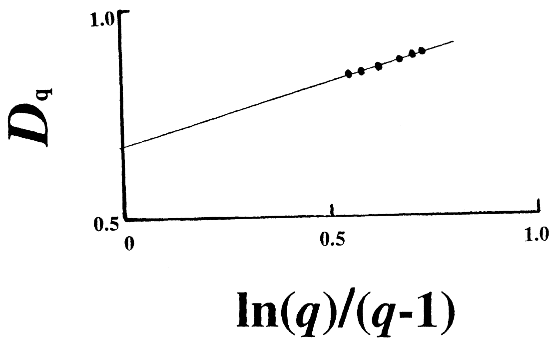

Fig. 3. The generalized dimensions D_q against $\ln q/(q-1)$ for experimental data obtained in a fluid motion at the onset of chaos (data taken from [14]). The straight line is drawn for comparison with CSH-approximation (4)

fluid wake of an oscillating cylinder at the onset of chaos (data taken from [14]). The straight line is drawn for comparison with CSH-approximation (4). The multifractal specific heat $c \simeq 0.27$ calculated from this Figure is approximately the same as those calculated for the hadron-hadron and for hadron-nucleus reactions.

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